Lecture Notes: 4-5 Curve Sketching (part 1)

GUIDELINES OF ALL CURVE SKETCHING PROBLEMS For each item below, write out in your own words how you actually find that item.

A. **Domain.** Find the domain of the function.

Look for "allowable" x-values avoiding ① zero in denominator, ② negative #'s under square root, ③ 0 or neg #'s in natural log, etc.

B. **Intercepts** Find any x- or y-intercepts.

x-intercept : set $y=0$. Solve for x

y-intercept : set $x=0$. Solve for y.

C. **Symmetry** Determine if the function is even or odd.

- use even powers or odd powers
- $\sin x$ is odd, $\cos x$ is even
- Some functions are neither

D. **Asymptotes** Identify any vertical or horizontal asymptotes.

$x=a$ is a vertical asymptote if $\lim\limits_{x \to a^{\pm}} f(x) = \pm\infty$

$y=b$ is a horizontal asymptote if $\lim\limits_{x \to \pm\infty} f(x) = b$

E. **Intervals of Increase or Decrease** Determine the intervals where the function is increasing and where the function is decreasing.

$f' > 0$ on I, then f is increasing on I

$f' < 0$ on I, then f is decreasing on I

F. **Local Maximum and Minimum Values** Identify any local maximums and minimums and where they occur.

If $f'(c)=0$ or $f'(c)$ is undefined and c is in the domain of $f(x)$, then $f(c)$ local max $\cap$ if f' is pos→neg; $f(c)$ local min $\cup$ if f' is neg→pos.

G. **Concavity and Points of Inflection** Find the intervals where the function is concave up and where the function is concave down. Identify any inflection points.

- $f'' > 0 \Rightarrow$ ccup $\cup$
- $f'' < 0 \Rightarrow$ ccdown $\cap$
- inflection point, (x,y), where concavity changes

H. **Sketch the Curve** Plot the curve. Include and label all the bits and pieces above.

- Include important points.

PRACTICE PROBLEM Sketch the curve $y = \frac{2x^2}{x^2-4} = \frac{2x^2}{(x+2)(x-2)}$

(a) Find the domain.

all real numbers except $x = \pm 2$. OR $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

(b) Find the x and y-intercepts.

If $x=0$, then $y = \frac{0}{-4} = 0$. If $y=0$, then $0 = \frac{2x^2}{x^2-4}$. So $x=0$.

Ans: x-intercept is 0, y-intercept is 0.

(c) Find the symmetries of the curve.

all powers are even.

answer: $f(x)$ is even

(d) Determine the asymptotes.

- Find the horizontal asymptotes.

$\lim_{x\to\infty} \frac{2x^2}{x^2-4} = 2$. $\lim_{x\to-\infty} \frac{2x^2}{x^2-4} = 2$. ANS: $y=2$

- Find the vertical asymptotes.

$\lim_{x\to -2^+} \frac{2x^2}{x^2-4} = -\infty$; $\lim_{x\to 2^+} \frac{2x^2}{x^2-4} = +\infty$ ANS $x=2$, $x=-2$

(e) Determine where the function is increasing/ decreasing.

$y = \frac{2x^2}{x^2-4}$

$y' = \frac{(x^2-4)(4x) - 2x^2(2x)}{(x^2-4)^2}$

$= \frac{4x^3 - 16x - 4x^3}{(x^2-4)^2}$

$= -\frac{16x}{(x^2-4)^2}$

critical pts: $x=0$

0

+++ 0 ---

sign of y'

ANSWER

f increases on $(-\infty, -2) \cup (-2, 0)$ and

decreases on $(0, 2) \cup (2, \infty)$

(f) Find the local maximum/ minimum values.

local max at $x=0$

max value is $f(0)=0$.

(g) Find the intervals of concavity/inflection points.

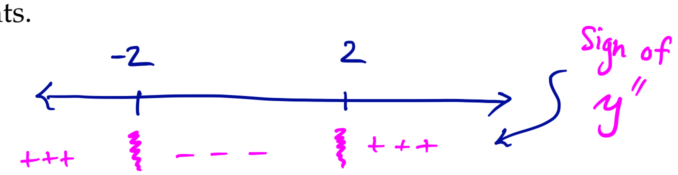
$$y' = \frac{-16x}{(x^2-4)^2}$$

$$y'' = \frac{(x^2-4)^2(-16) + 16x \cdot 2(x^2-4)^1(2x)}{(x^2-4)^4}$$

$$= \frac{16(x^2-4)\left[-(x^2-4) + 4x^2\right]}{(x^2-4)^4}$$

$$= \frac{16\left[3x^2+4\right]}{(x^2-4)^3}$$

← numerator never zero! alway positive



ANS:

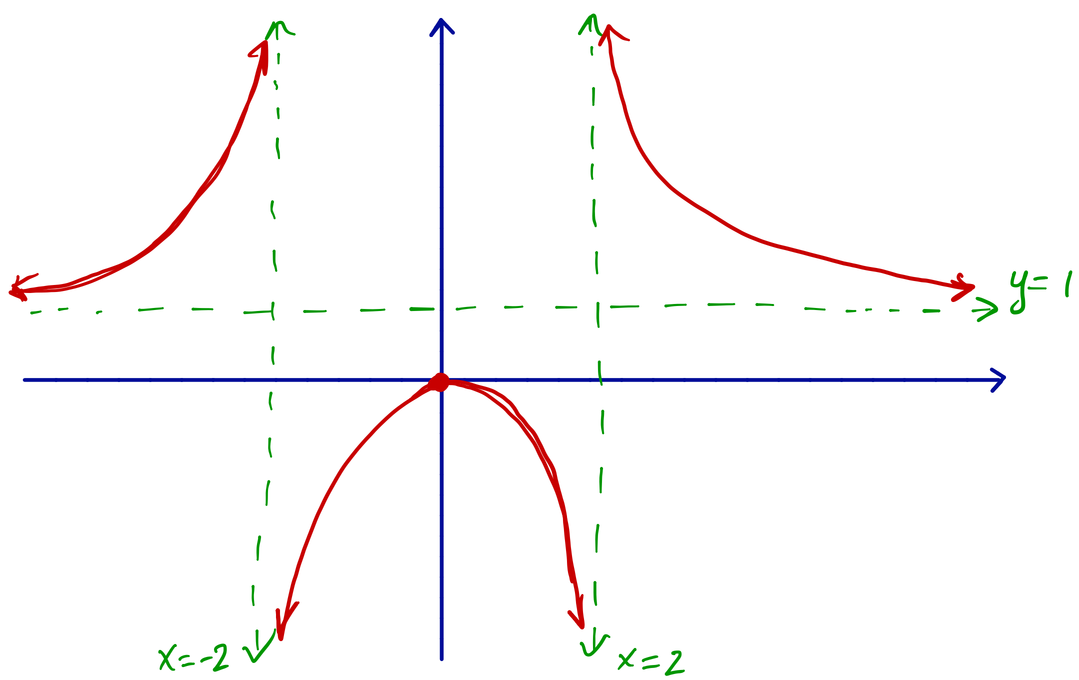
f is concave up on $(-\infty,-2)\cup(2,\infty)$

and concave down on $(-2,2)$

(h) Sketch the curve.

plot important points

$(0,0)$



★ Check your answers using a graphing device!